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DETERMINATION OF THE FREQUENCY OF AN OSCILLATOR WITH A CONDENSER

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ABSTRACT. With an a. c. galvanometer of the form of a watt-meter or a dynamometer null point is obtained under two conditions, namely, (1) when the current through either the fixed or the moving coil is zero and (2) when the phase difference between the currents in the two systems of coils is 90°. In the method described below, advantage is taken of the second condition The resistances and the condenser are arranged in the arms of a Wheatstone net, the moving coil in one of the diagonal arms and the fixed coils in series with the current source in the other diagonal arm. It is shown that the phase-difference between the currents in the two systems of coils is 90° when

$$\frac{1}{C^2p^2}=\frac{Q}{P}\,R\left\{R+\frac{G(P+Q)}{P+Q+G}\right\}.$$

INTRODUCTION

When a dynamometer or a watt-meter is used as a recording instrument for alternating currents, the deflection of its moving coil is given by

$$\delta = A.\ I_f I_m \cos\chi$$

where I_f, I_m are the R. M. S. values of the currents in the fixed and moving coils respectively and χ, the phase difference between them. It is obvious that the deflection is one way or the other according as $\cos\chi$ is positive or negative and is zero when (1) I_f or $I_m=0$ or (2) $\chi=\frac{\pi}{2}$. In experiments which require the fulfilment of the first condition such an instrument can be used in place of telephones. In the experiment described below, however, the null point is due to the fulfilment of the second condition.

THEORY

A system consisting of a resistance R and a reactance X acts with respect to an alternating current as an impedance of value $R+jX$. If the reactance is inductive. $X=Lp$, where L=inductance and $p=2\pi$.frequency ; if the reactance is capacitative, $X=-\frac{1}{Cp}$, where C=capacity.

In figure 1, the impedances of the arms ab, bc and ad consist of the non-inductive resistances P, Q and R respectively while the impedance of the arm cd is $-\dfrac{j}{Cp}$. The arm ac contains the moving coil of an a.c. galvanometer of the form of a vertical watt-meter or a dynamometer. Assuming the inductance of the moving coil to be zero, the impedance of this arm is the resistance, G. Let i_1, i_2, etc., be the currents in the different parts of the network at any instant. Then

$$\left.\begin{aligned} i &= i_1 + i_3 \\ i_2 &= i_1 - i_g \\ i_4 &= i_3 + i_g \end{aligned}\right\} \quad \ldots \quad (1)$$

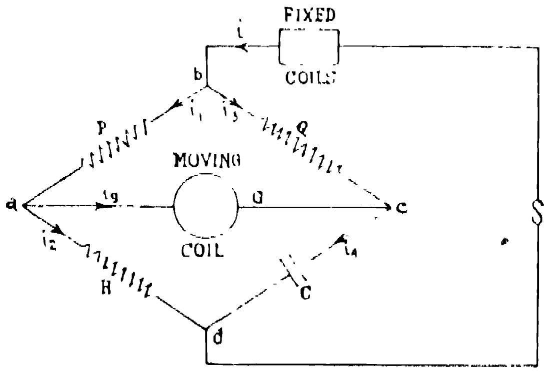

FIGURE 1.

In the mesh $abca$

$$Gi_g - Qi_3 + Pi_1 = 0 \quad \ldots \quad (2)$$

and in the mesh $bcdb$

$$Gi_g - \frac{j}{Cp} i_4 - Ri_2 = 0$$

or

$$(R + G + jX)i_g + jXi_3 - Ri_1 = 0 \quad \ldots \quad (3)$$

where

$$X = -\frac{j}{Cp}$$

From (1), (2) and (3)

$$\frac{i_g}{QR - jPX} = \frac{i_3}{P(R+G+jX)+RG} = \frac{i_1}{Q(R+G+jX)+jXG}$$

$$= \frac{i}{R(P+Q+G)+G(P+Q)+jX(P+Q+G)}$$

or
$$\frac{i_{ii}}{P\left(\frac{Q}{P}R-jX\right)}=\frac{i}{(P+Q+G)\left\{R+\frac{G(P+Q)}{P+Q+G}+jX\right\}}$$

or
$$\frac{i_{ii}}{i}=\frac{P}{P+Q+G}\cdot\frac{\sqrt{\frac{Q^2}{P^2}R^2+X^2}}{\sqrt{\left\{R+\frac{G(P+Q)}{P+Q+G}\right\}^2+X^2}}\,\epsilon^{-j(\phi+\theta)}\qquad\ldots\quad(4)$$

where
$$\tan\phi=\frac{X}{\frac{Q}{P}R}$$

and
$$\tan\theta=\frac{X}{R+\frac{G(P+Q)}{P+Q+G}}\qquad\ldots\quad(5)$$

Thus the phase difference between i_{ii} and i is $\phi+\theta$. The deflection of the moving coil will, therefore, be zero if

$$\phi+\theta=\frac{\pi}{2}$$

or
$$\tan\phi=\cot\theta$$

or
$$\frac{X}{\frac{Q}{P}R}=\frac{R+\frac{G(P+Q)}{P+Q+G}}{X}$$

or
$$X^2=\frac{1}{C^2p^2}=\frac{Q}{P}R\left\{R+\frac{G(P+Q)}{P+Q+G}\right\}\qquad\ldots\quad(6)$$

In deducing the above equation, the inductance of the moving coil has been assumed to be zero. In practice, however, that is not the case. The inductance should therefore, be compensated by a preliminary experiment from which the resistance, G, of the moving coil arm will also be obtained. The connections for the purpose are given in figure 2.

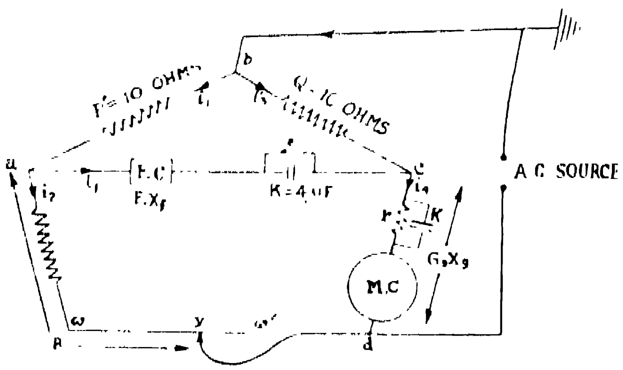

FIGURE 2.

If F and G are the resistances, X_f and X_g, the reactances of the arms ac and cd respectively,

$$P'i'_1+(F+jX_f)i'_f-Q'i'_3=0$$

or
$$(P'+Q'+F+jX_f)i'_f-Q'i'_3+P'i'_2=0$$

and
$$(F+jX_f)i'_f+(G+jX_g)i'_4-R'i'_2=0$$

whence
$$\frac{i'_f}{Q'R'-P'G-jP'X_g}=\frac{i'_4}{R'(P'+Q')+F(P'+R')+jX_f(P'+R')}$$

The phase difference between i'_f and i'_4 is $\theta'+\phi'$, where

$$\tan\theta'=\frac{X_f}{F+\dfrac{R'(P'+Q')}{P'+R'}}$$

and
$$\tan\phi'=\frac{X_g}{\dfrac{Q'}{P'}R'-G}$$

There will be no deflection if $\theta'+\phi'=\dfrac{\pi}{2}$, or $\tan\phi'=\cot\theta'$

or
$$X_fX_g=\left(\frac{Q'}{P'}R'-G\right)\left\{F+\frac{R'(P'+Q')}{P'+R'}\right\} \qquad \dots (7)$$

There will also be no deflection if $i'_f=0$, the conditions for which are

$$\left.\begin{aligned}\frac{Q'}{P'}R'&=G\\ \text{and}\quad X_g&=0\end{aligned}\right\} \qquad \dots (8)$$

It is easy to see that

$$X_g = L_g P - \frac{r^2 k p}{1 + r^2 k^2 p^2}$$

where $L_g =$ inductance of the moving coil, so that

when $X_g = 0$

$$L_g = \frac{r^2 k}{1 + r^2 k^2 p^2} \tag{9}$$

EXPERIMENT

(*i*) *Arrangement for securing constancy of frequency* :—Since in most cases the frequency of an audio oscillator depends on the strength of the current drawn in the experimental circuit, the device shown in figure 3, may be adopted to ensure constancy of the frequency.

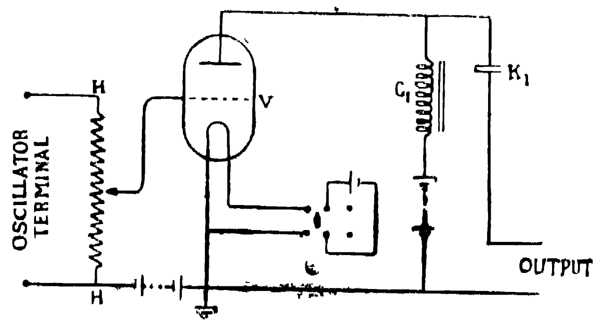

FIGURE 3.

V⟶Valve having an internal resistance less than 300 ohms. It may be of type Philips E 406 or Mazda Ac/P1.

HH⟶High resistance potentiometer.

C_1⟶Low frequency choke of valve 20 to 100 henries.

K_1⟶Condenser of capacity 2 to 8 μF.

t⟶Mercury cup key intended to let the output current flow only as long as necessary.

(*ii*) *Adjustment of the position of the moving coil* :—When using a galvanometer of the watt-meter or dynamometer type for a. c. measurements, the initial position of the moving coil should be carefully adjusted. If the position is not symmetrical with respect to the fixed coils, there will be a linking between the moving coil and the lines of force due to a current in the fixed coils and if this current is alternating, there will be an induced current in the moving coil tending to bring it to the symmetrical position. To start with, therefore, the levelling screws and the torsion head should be adjusted so that the moving coil may be symmetrically placed with respect to the fixed coils. The former is then short-circuited and a fairly high alternating current sent through the latter. If there is a deflection, the torsion-head should be turned in the direction of the deflection till the moving coil remains unaffected by the current in the fixed coils.

(iii) *Compensation of the moving coil* :—The arrangement of the apparatus is shown in figure 2. The balance corresponding to equation (7) is easily obtained for any value of r simply by adjusting R'. After obtaining such a balance, the condenser K is short-circuited to alter the phase difference between i'_r an i'_i. The balance will be disturbed. The procedure is repeated with different values of r till there is no deflection whether K is short-circuited or not, *i.e.*, till the balance is independent of the reactance of the arm *ac*. The co-efficient of X_r must then be zero, *i.e.*,

$$X_{ll} = 0$$

and therefore, (since $Q' = P'$)

$$R' = G.$$

The resistance of the part, $w\ y$, together with that of the connecting wires in the R′-arm is determined in the manner described in (V).

(iv) *Arrangement for the Determination of Frequency* :—The fixed coil and the compensated moving coil are now connected as in figure 4, and with the standard condenser C in the fourth arm, *cd*, of the network, R is adjusted till balance.

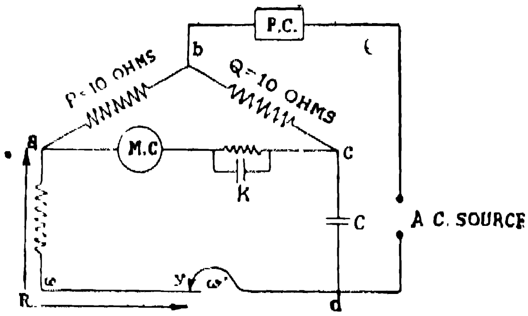

FIGURE 4.

(*v*) *Determination of the Unknown Parts of theResistance of the Arm ad* :—

The unknown parts of the resistance of the R-arm (fig. 4) or the R′-arm (fig. 2) are the resistance of the connecting wires and that of the wire $w\,w'$ between the end w and the final balance point y. In order to determine the total value of these resistances, the *a. c.* souice is replaced by a *d. c.* source in the last arrangement and the ratio Q/P made equal to 1.0. The known iesistances in the part $a\,w$ are all plugged. Instead of the condenser, C, a resistance box is put in the arm, *c d*, and the resistance of this arm adjusted for balance at the final null points botained in (*iii*) and (*iv*).

REMARKS

In selecting the values of the resistances P and Q for experiment (*iv*) two points are to be borne in mind.

(*a*) It will be noticed from equation (4) that the ratio i_g/i depends on the fraction $\frac{P}{P+Q+G}$. This fraction should, therefore, not be too small, otherwise the sensitivity of the arrangement will be affected.

(*b*) From equation (6) it will be seen that the accuracy of the result depends among other things on the quantity $\frac{G(P+Q)}{P+Q+G}$. Now G is determined by a separate experiment ; hence in order that any error introduced in its determination may not appreciably affect the result, (P+Q) should be suitably small in compaiison with G.

In the experiment conducted in this laboratory, the value of G after the compensation of the moving coil was 136 ohms. Hence keeping the two points mentioned above in view P and Q were each made equal to 10 ohms.

It is obvious that the oscillator used must give a sinuous *e.m.f.* oi be one in which the harmonics have a very low intensity. A Vreeland oscillator or a carefully constructed valve oscillator is very suitable.

It is also possible to detcimine the frequency with the help of an inductance instead of a capacity—the formula in this case will be

$$L^2p^2 = \left\{\frac{Q}{P}\,R - S\right\}\left\{R+S+\frac{G(P+Q)}{P+Q+G}\right\}$$

where L = inductance and S = resistance of the arm containing it. But in this case, unless L is fairly large (*i.e.*, about 10 *m. h.* or more), $\left(\frac{Q}{P}\,R-S\right)$ is small and so the error in the determination is comparatively large.

The method of compensating an inductance described above is known as Sumpner's method. The effective resistance of the system consisting of the condenser, k, and the resistance, r, (fig. 2) is $\frac{r}{1+r^2k^2p^2}$ and the effective reactance $-\frac{r^2kp}{1+r^2k^2p^2}$. Thus by choosing a suitable condenser, the compensation can be made practically independent of the frequency of the oscillator up to 1000 cycles per sec. The inductance of the moving coil of the galvanometer used in this laboratory was about 0·36 m.h. The condenser chosen had, therefore, a capacity of 0·03 μF.

Once the moving coil is carefully compensated, the galvanometer can be used to determine frequencies covering a wide range within error of 1 in 3000. Conversely if the frequency is known, the method affords means of determining capacities quickly and accurately.

For determination of inductances, the method is more convenient than Wien's method in that it entails an economy of apparatus and time—provided that the compensating system is regarded as a part of the galvanometer—while the accuracy of the result obtained is not less.

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